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HEAT TRANSFER WITH CONTACT RESISTANCE

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Abstract—The thermal contact resistance of two solids touching each another was analyzed with particular reference to the shape of a single heat channel. This channel was assumed to have a cylindrical contour whose radius near the interface decreases gradually to the contact area forming a truncated cone.

The contact resistance of the interface was found as a function of the cone angle, the ratio of the radii of the truncated cone and the properties of the materials involved.

From a numerical solution it was found that the contact resistance can be described by the properties of the materials, the number of contact areas and a single function of the radii ratio and that, for small values of $\cot\gamma$, the contact resistance is almost insensitive to the cone angle.

NOMENCLATURE

a,	radius of contacting circle;
A_i,	coefficient (equation 20);
c,	radius of heat channel;
$f_I(r)$,	$= q/\pi a^2$ (equation 24);
$f_{II}(r)$,	$q/2\pi a\sqrt{(a^2-r^2)}$ (equation 25);
$f(\varepsilon)$,	function of ε (equation 6);
J_0, J_1,	Bessel functions;
k,	thermal conductivity;
k_s,	$= k_1k_2/(k_1+k_2)$ combined thermal conductivity;
L,	length (far away from the contact);
M,	number of divisions;
N,	number of contacts;
q,	heat transfer rate;
r,	radial coordinate;
R,	total contact resistance;
s,	a coordinate (Fig. 2);
T,	temperature;
T_0	reference temperature at $z = L$;
U,	temperature difference;
$\bar{U}$,	average temperature difference;
u,	a coordinate (Fig. 2);
z,	axial coordinate;
ΔT_c,	total interface temperature drop;
δ,	height of contact cylinder;
η,	exponent (equation 12);
ε,	ratio, $\varepsilon = a/c$;
λ_i,	eigenvalue;
γ,	angle (Fig. 2);
$\psi(\varepsilon)$,	function of ε (equation 8);
$\mu(\varepsilon)$,	function of ε (equation 29);
$\xi(\cot\gamma)$,	function of $\cot\gamma$ (equation 29);
$\phi(\varepsilon)$,	function of ε (equation 11).

Subscripts

1, 2,	metals 1 and 2 respectively, in contact;
i, j,	variable index.

1. INTRODUCTION

WHEN TWO solids having rough flat surfaces are touching one another, a metallic contact is formed at a discrete number of contact areas, whose total area is smaller than the cross-section of each of the two bodies. The number and size of the contact areas depend upon the applied external force between the surfaces and their mechanical properties.

If heat (or electricity) is transferred from one body to the other, an additional thermal resistance exists at the interface. This thermal resistance exhibits a temperature drop at the interface when heat flows, at steady state, normally to the interface. The temperature drop at the interface is the temperature difference obtained by extrapolating the temperature profile of the two bodies to the interface.

The overall thermal resistance is found by dividing the temperature drop by the heat flow:

$$R = \frac{\Delta T_c}{q}. \tag{1}$$

However, in trying to analyze the thermal resistance, one has to examine the individual thermal resistance R_j at any contact area spot, from which the total thermal resistance is derived:

$$\frac{1}{R} = \sum_{j=1}^{N} \frac{1}{R_j} \tag{2}$$

where N is the number of contacting areas.

In the last twenty years, a growing number of works have been published on thermal contact resistance. Most of them are experimental works, but quite a few are theoretical [1–6]. Basically, in order to analyze thermal contact resistance, one can divide the problem into three parts: surface analysis, deformation analysis and thermal analysis.

In surface analysis, surfaces are assumed to be rough and flat, the roughness being random with a Gaussian distribution of heights [4]. Both surface analysis and deformation analysis provide means of calculating the number of contacts per unit area and their average radius of the contact area (assuming uniform circular areas), as functions of surface roughness, surface profile slope, the mechanical properties of the two bodies in contact and applied external force.

The thermal analysis of a single heat channel was treated in various ways. Fenech *et al.* [1] assumed that all the heat channels have the same size and are evenly distributed, each having a cylindrical shape of radius c. At the interface between the two solids, the contact point consists of small cylinders of radius a, and heights δ_1 and δ_2, respectively, for two solids 1 and 2. (Fig. 1a).

The contact area of each solid was divided into three zones for which the temperature distribution is found by solving the three differential equations and the interconnected boundary conditions. The interface contact resistance was calculated by extrapolating the temperature profiles to the interface and using equation (1). For the case of a vacuum in the interface, the thermal resistance is

$$R_j = \frac{(1-\varepsilon^2)\left[\dfrac{2.4\delta_1+a}{ak_1}+\dfrac{2.4\delta_2+a}{ak_2}\right]}{2.4\,\pi a} \tag{3}$$

and if we assume $\delta_1 = \delta_2 = \delta$, equation (3) can be written as:

$$R_j = \frac{4}{\pi k_s a}\left[(1-\varepsilon^2)\left(\frac{2.4\delta + a}{9.6a}\right)\right]. \tag{4}$$

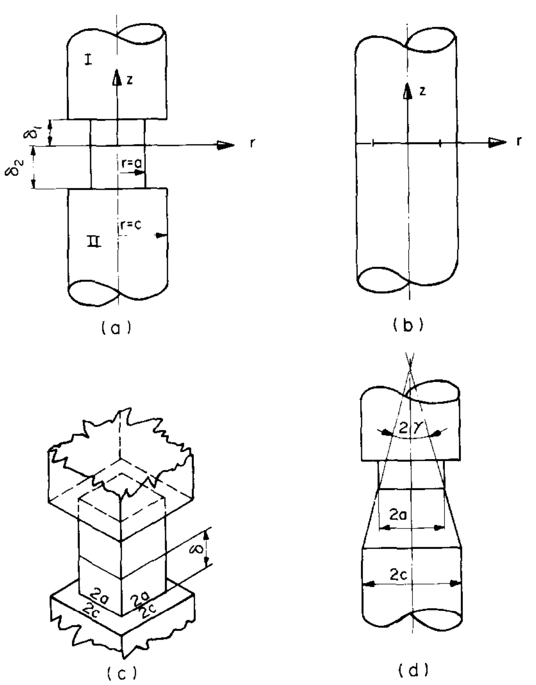

FIG. 1. Heat channels; (a) Fenech [1]; (b) Mikic [4]; (c) Bochorishvili [7]; (d) Williams [9].

Mikic *et al.* [4] assumed semi-infinite cylindrical heat channels of radius c, where heat is supplied over a circular area of radius a ($a<c$), at the base of the cylinder ($z = 0$) (Fig. 1b). This model is similar to Fenech's, except that the small cylinders of radius a have $\delta_1 = \delta_2 = 0$.

By choosing such a geometry, the authors could obtain a closed form solution for the temperature distribution along the heat channel, from which the thermal resistance was calculated. Their results for different thermal conditions such as constant temperature or uniform heat flux at the base, without fluid at the interface, can be written as:

$$R_j = \frac{4}{\pi k_s a} f(\varepsilon) \tag{5}$$

where $f(\varepsilon)$ varies very little for the different thermal conditions. For small values of ε:

$$f(\varepsilon) = \frac{\pi}{16} - \frac{\varepsilon}{4}. \tag{6}$$

Bochorishvili *et al.* [7] made use of the solution of Smythe [8] which was derived for the electrical resistance of a plate with variable cross-section, using the analogy between Fourier's and Ohm's laws. For the case of a small square cross-section of length a that forms the contact and of the distance c between contacts (see Fig. 1c), they found that:

$$R_j = \frac{4}{\pi k_s a}\psi(\varepsilon), \tag{7}$$

where

$$\psi(\varepsilon) = \frac{1}{16}\left[\left(\varepsilon + \frac{1}{\varepsilon}\right)\ln\left(\frac{1+\varepsilon}{1-\varepsilon}\right) + 2\ln\left(\frac{1-\varepsilon^2}{4\varepsilon}\right)\right]. \tag{8}$$

Williams [9] treated, analytically and experimentally, a single point contact, formed between a cylindrical heat channel ended by a cone, and a flat plane. He assumed that, when the tip of the cone is pressed against a hard flat plane, it is deformed to a small cylinder. The volume of this small cylinder (smaller than the basic cylinder) is the same as that of the tip.

The thermal resistance of the deformed cone is the sum of the resistances of the truncated cone and the small cylinder minus the resistance of the basic cylinder extended to the interface. Using Fig. 1(d), and a 1-dim. model (spherical isotherms in the truncated cone), he obtained

$$R_j = \frac{(1-\varepsilon)\left[3+(5-4\varepsilon)\cos\gamma\right]}{6\pi k_s a \sin\gamma}. \tag{9}$$

Yovanovich [10] extended the work of Mikic [4] on thermal constriction resistance of coaxial cylindrical contacts. By selecting an arbitrary heat flux distribution, $f(r/a)$, at the contacting surface, he developed a general expression for determining the thermal resistance, as follows:

$$R_j = \frac{4}{\pi k_s a}\phi(\varepsilon) \tag{10}$$

where

$$\phi(\varepsilon) = \frac{1}{2}\frac{\sum_{i=1}^{\infty}\frac{J_1(\lambda_i c)}{(\lambda_i c)^2 J_0^2(\lambda_i c)}}{\int_0^a rf\left(\frac{r}{a}\right)\mathrm{d}r}\int_0^a rf\left(\frac{r}{a}\right)J_0(\lambda_i r)\,\mathrm{d}r. \tag{11}$$

The heat flux distribution was assumed to be proportional to

$$f\left(\frac{r}{a}\right) = \left[1-\left(\frac{r}{a}\right)^2\right]^\eta \tag{12}$$

where η had the values of $\eta = -\frac{1}{2}, 0, \frac{1}{2}$. His [10] results were presented in a tabulated form.

It is the purpose of this work to find analytically, the sensitivity of the shape of the heat channel on the thermal contact resistance of a single heat channel.

2. THERMAL ANALYSIS

The analysis is based on a single heat channel of a cylindrical contour, where the radius of the channel near the contact area decreases gradually to the contacting radius (Fig. 2) [11].

It is assumed that heat is transferred only at the contact area so that radiation effects and conduction through fluid which might exist at the interface are neglected.

The calculation is performed on one body from which the temperature distribution and thermal resistance will be derived. The total resistance of the contact will be the sum of two resistances in series.

2.1. *Temperature distribution*

The conduction heat transfer at steady state in cylindrical coordinates (no heat sources) is given by:

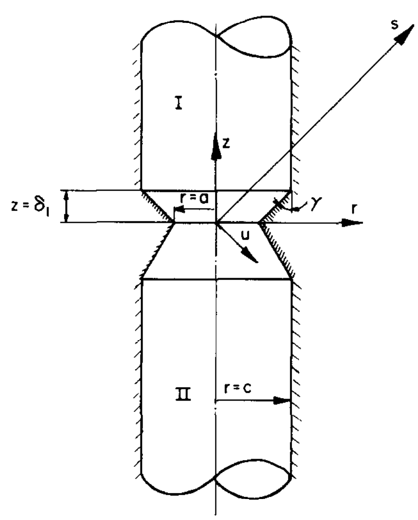

FIG. 2. Heat channel for thermal analysis in this work.

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} = 0. \tag{13}$$

In order to describe the boundary conditions at the truncated cone it is worthwhile to define another set of coordinates as follows:

$$u = r\cos\gamma - z\sin\gamma \tag{14a}$$

$$s = r\sin\gamma + z\cos\gamma. \tag{14b}$$

With the above coordinates, the boundary conditions are as follows:

$$0<r<a \qquad z=0 \qquad -k\left(\frac{\partial T}{\partial z}\right) = f(r), \tag{15a}$$

$$a<r<c \qquad 0<z<\delta \qquad \frac{\partial T}{\partial u} = 0, \tag{15b}$$

$$r=0 \qquad z>0 \qquad \frac{\partial T}{\partial r} = 0, \tag{16}$$

$$r=c \qquad z>\delta \qquad \frac{\partial T}{\partial r} = 0, \tag{17}$$

$$r>0 \qquad z = L \gg \delta \qquad -k\left(\frac{\partial T}{\partial z}\right) = \frac{q}{\pi c^2}. \tag{18}$$

We define the reference temperature T_0 so that $T_{z=L} = T_0$.

If

$$U = T - T_0 \tag{19}$$

the solution of (13) with the boundary conditions (16) and (18) gives:

$$U = \frac{q}{\pi k c^2}(L-z) + \sum_{i=1}^{\infty} A_i e^{-\lambda_i z} J_0(\lambda_i r) \tag{20}$$

where, from (17)

$$J_1(\lambda_i c) = 0, \tag{21}$$

from which the eigenvalues λ_i can be obtained.

To find the coefficient A_i, we make use of equations (15a) and (15b) and get:

$$\sum_{i=1}^{\infty} A_i\lambda_i J_0(\lambda_i r) = \frac{f(r)}{k} - \frac{q}{\pi k c^2}, \quad 0<r<a \tag{22}$$

$$\sum_{i=1}^{\infty} A_i\lambda_i e^{-\lambda_i(r-a)\cot\gamma}\left[J_1(\lambda_i r)\cot\gamma - J_0(\lambda_i r)\right] = \frac{q}{\pi k c^2} \quad a<r<c. \tag{23}$$

The function $f(r)$ can have various forms, for instance, a uniform heat flux at the contact:

$$f_1(r) = \frac{q}{\pi a^2}. \tag{24}$$

A similar function which describes approximately constant temperature at the interface is

$$f_{II}(r) = \frac{2}{2\pi a\sqrt{(a^2-r^2)}}. \quad (25)$$

Without losing the generality of the solution, equation (24) is used for further calculations. Equation (22) therefore becomes

$$\sum_{i=1}^{\gamma} A_i\lambda_i J_0(\lambda_i r) = \frac{q}{\pi k a^2}\left(1 - \frac{a^2}{c^2}\right) \quad 0 < r < a. \quad (26)$$

Equations (23) and (26) provide means of calculating the values A_i, and together with equation (20) form the solution of the problem, giving the temperature distribution in the body.

Unfortunately, the values A_i cannot be found analytically. Therefore, a numerical computer was used to find A_i and the temperature profile. The method used is described in Section 3.

2.2. *Contact resistance*

As stated before, the contact resistance of each body is defined as the temperature drop at the interface, divided by the heat transfer.

From Fig. 3, one can see the temperature drop ΔT_1 due to the contact resistance, whose division by the heat gives the contact resistance:

$$R_1 = \frac{\bar{U}_{z=0} - \left(\bar{U}_{z=\delta} + \frac{q}{\pi k c^2}\delta\right)}{q}. \quad (27)$$

After using equation (20) to find the average temperature, equation (27) gives:

$$R_1 = \frac{2}{qa}\sum_{i=1}^{\infty}\frac{A_i J_1(\lambda_i a)}{\lambda_i}. \quad (28)$$

Here again, the knowledge of the values of A_i will give the value of R_1, which can be presented, in general by:

$$R_1 = \frac{4}{\pi k_1 a}\mu(\varepsilon)\,\xi(\cot\gamma), \quad (29)$$

where $\mu(\varepsilon)$ is a function of the ratio of the contact radius to the radius of the heat channel and $\xi(\cot\gamma)$ is a function of the cone's angle.

The contact resistance of the two bodies with the same geometry but with a different thermal conductivity will have the form:

$$R_j = (R_1 + R_2) = \frac{4}{\pi k_s a}[\mu(\varepsilon)\,\xi(\cot\gamma)]. \quad (30)$$

3. NUMERICAL SOLUTION

The values of A_i in equations (23) and (26) cannot be found analytically. Therefore a numerical solution is used. The numerical procedure is to divide the truncated cone of the contacting interface into M divisions of equal radial spaces.

$$\Delta r_i = \frac{c}{M}. \quad (31)$$

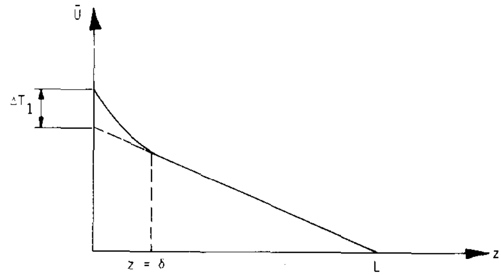

FIG. 3. Average temperature distribution as function of z.

The selection of the number M will determine the number of series members of equation (20), and therefore the accuracy of the solution.

From the boundary conditions (15a), (15b), equations (23) and (26) were obtained respectively. The substitution of the values of r_i for each division will provide M linear equations with M unknown A_i.

Putting the numerical values of A_i into equation (20) and performing the summation we obtain the numerical values of the temperature for each point of the contact area. However, using the same values of A_i in equation (28) we get the contact resistance of one body. From equations (23), (26) and (28), it follows that the contact resistance of a single channel depends upon three parameters: one which depends upon the properties of the materials in contact, another one which depends upon the ratio of the contact radius to the radius of the heat channel, and finally one which is related to the angle of the truncated cone. These three parameters can thus be presented as in equation (29), namely,

$$R_1 = \frac{4}{\pi k_1 a}\mu(\varepsilon)\,\xi(\cot\gamma).$$

Using the numerical solution, the function $\xi(\cot\gamma)$ is found numerically (Fig. 4). The function $\mu(\varepsilon)$ is presented in Fig. 5 for the case of constant heat flux and uniform temperature in the interface.

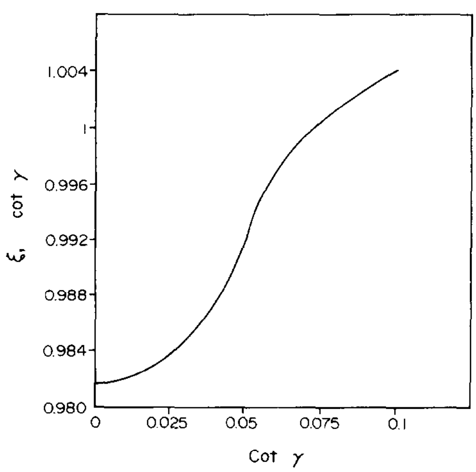

FIG. 4. Variation of ξ vs $\cot\gamma$.

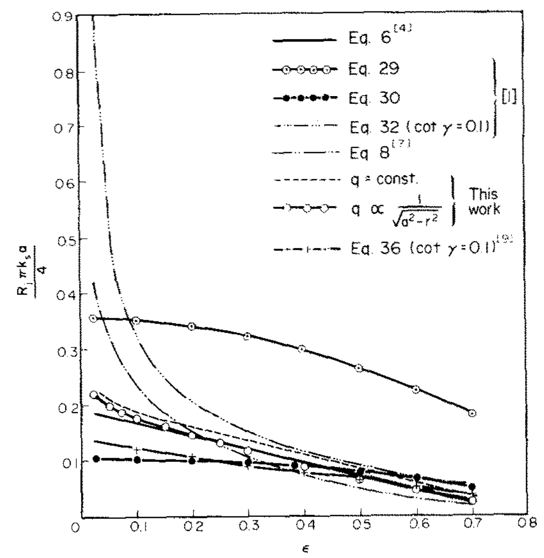

FIG. 5. Contact resistance as function of ε.

4. DISCUSSION

Most analytical works dealing with the contact resistance of a single heat channel, present their results as a function of ε, whereas the shape of the contact area is not treated. Mikic [4] assumes a flat circular contact area, which means $\cot\gamma = 0$, and obtains a single function $f(\varepsilon)$, equation (6). Bochorishvili [7], also, obtain a single function $\psi(\varepsilon)$, equation (8).

Fenech's [1] results include a function of ε and another function of the length of the contacting cylinders (equation 4). If we assume that the length δ is similar to that assumed by Bochorischvili, namely $\delta = a$, we get:

$$R_j = \frac{4}{\pi k_s a}[0.354(1-\varepsilon^2)]. \tag{32}$$

If we assume, like Mikic, $\delta = 0$,

$$R_j = \frac{4}{\pi k_s a}[0.104(1-\varepsilon^2)]. \tag{33}$$

However, assuming that δ is related to a through the angle γ, so that

$$\delta = (c-a)\cot\gamma \tag{34}$$

we get:

$$R_j = \frac{4}{\pi k_s a}\left[\frac{2.4\cot\gamma + (1-2.4\cot\gamma)\varepsilon}{9.6\varepsilon}(1-\varepsilon^2)\right]. \tag{35}$$

The shape of the contact in Williams [9] analysis, is similar to this work if one ignores the existence of the small cylinder. Therefore, according to Williams [9], the thermal resistance of a contact having the shape of a truncated cone can be found as

$$R_j = \frac{4}{\pi k_s a}\,\frac{(1-\varepsilon)[1+(1-2\varepsilon)\cos\gamma]}{8\sin\gamma}. \tag{36}$$

Equations (6), (8), (32), (33), (35) and (36), with $\cot\gamma = 0.1$, are presented in Fig. 5, for comparison with this work.

The results of Yovanovich [10] agree well with those of Mikic [4] for the cases of $\eta = -\frac{1}{2}, 0$.

From this work and Fig. 4 it seems that the function $\xi(\cot\gamma)$ is almost insensitive to $\cot\gamma$ being very close to unity for small values of $\cot\gamma$, therefore, one should expect a satisfactory description of the contact resistance by a single function of ε as presented in Fig. 5. From Fig. 5, it seems that this work is in good agreement with Mikic [4] rather than with Fenech, Bochorischvili or Williams. Therefore, the contact resistance of a single heat channel is better described by a resistance of a flat circular surface rather than by contacting cylinders.

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TRANSFERT THERMIQUE AVEC RESISTANCE DE CONTACT

Résumé—On analyse la résistance thermique de contact de deux solides en contact, en considérant particuliérement un seul canal pour la chaleur. Ce canal est supposé avoir un contour cylindrique dont le rayon près de l'interface décroît graduellement jusqu'à celui de l'aire de contact pour former un tronc de cône.

La résistance de contact est une fonction de l'angle du cône, du rapport des rayons du tronc de cône et des propriétés des matériaux.

A partir d'une solution numérique, on trouvre que la résistance de contact peut être décrite par les propriétés des matériaux, le nombre des aires de contact et par une seule fonction du rapport des rayons et que pour les petites valeurs de cotg γ, la résistance de contact est presque insensible à l'angle du cône.

WÄRMEÜBERGANG MIT KONTAKTWIDERSTAND

Zusammenfassung—Der thermische Kontaktwiderstand zwischen zwei einander berührenden Feststoffen wurde unter besonderer Berücksichtigung der Form eines einzelnen Wärmeströmungskanals untersucht. Für diesen Kanal wurde eine zylindrische Form angenommen, deren Radius in der Nähe der Grenzfläche allmählich in Richtung der Kontaktfläche abnimmt, so daß ein stumpfer Kegel entsteht. Der Kontaktwiderstand der Grenzfläche ergab sich als eine Funktion des Kegelwinkels, des Radienverhältnisses des stumpfen Kegels und der Stoffwerte der beteiligten Marterialien. Mit Hilfe einer numerischen Lösung wurde gefunden, daß der Kontaktwiderstand sich durch die Stoffwerte der Materialien, die Anzahl der Kontaktflächen und eine einzige Funktion des Radienverhältnisses beschreiben läßt und daß für kleine Werte von ctg γ der Kontaktwiderstand nahezu unabhängig vom Kegelwinkel ist.

ТЕПЛОПЕРЕНОС ПРИ КОНТАКТНОМ СОПРОТИВЛЕНИИ

Аннотация — Проведен анализ теплового контактного сопротивления двух соприкасающихся твердых тел, причем особое внимание обращено на форму единичного теплового канала. Предполагается, что канал имеет цилиндрический контур, радиус которого у границы раздела постепенно уменьшается, так что поверхность контакта является основанием усеченного конуса. Найдено, что контактное сопротивление границы раздела является функцией угла вершины конуса, отношения радиусов усеченного конуса и свойств используемых материалов. Путем численного решения показано, что контактное сопротивление можно описать с помощью характеристик материалов, числа контактных областей и единичной функции отношения радиусов, а также, что при небольших значениях cot γ контактное сопротивление почти не зависит от угла конусности.